Integrationsverfahren (PI und Sub)

Lösungen

1.) PI oder ausmultiplizieren

$$\int \underset{v'}{x} \cdot \underset{u}{(x-1)}\, dx = \Big[\underset{v}{\tfrac{1}{2}x^2}\underset{u}{(x-1)}\Big] - \int \underset{v}{\tfrac{1}{2}x^2} \cdot \underset{u'}{1}\, dx$$

$$= \Big[\tfrac{1}{2}x^2(x-1)\Big] - \Big[\tfrac{1}{6}x^3\Big]$$

2.) PI oder ausmultiplizieren

$$\int \underset{u}{(x-2)} \cdot \underset{v'}{x^2}\, dx = \Big[\underset{u}{(x-2)} \cdot \underset{v}{\tfrac{1}{3}x^3}\Big] - \int \underset{u'}{1} \cdot \underset{v}{\tfrac{1}{3}x^3}\, dx$$

$$= \Big[(x-2)\,\tfrac{1}{3}x^3\Big] - \Big[\tfrac{1}{12}x^4\Big]$$

3.) (Linear Substitution) oder ...

$$\int e^{2x+3}\, dx = \Big[\tfrac{1}{2}e^{2x+3}\Big]$$

4.) Sub

$$\int x \cdot \sqrt{2x^2-5}\, dx = \tfrac{1}{4}\int 4x\, \sqrt{2x^2-5}\, dx$$

$$z = g(x) = 2x^2-5 \quad , \quad g'(x) = 4x$$

$$f'(z) = \sqrt{z} \qquad f(z) = \tfrac{2}{3}z^{3/2}$$

$$= \tfrac{1}{4}\Big[\tfrac{2}{3}(2x^2-5)^{3/2}\Big]$$

5.) PI

$$\int \underset{u}{x} \cdot \underset{v'}{e^x}\, dx = \Big[\underset{u}{x} \cdot \underset{v}{e^x}\Big] - \int \underset{u'}{1} \cdot \underset{v}{e^x}\, dx$$

$$= \Big[x e^x\Big] - \Big[1 \cdot e^x\Big]$$

6.) PI mit Trick?

$$\int \ln x \, dx = \int \underset{v'}{1} \cdot \underset{u}{\ln x} \, dx$$

$$= [\underset{v}{x} \cdot \underset{u}{\ln x}] - \int \underset{v}{x} \cdot \underset{u'}{\tfrac{1}{x}} \, dx$$

$$= [x \cdot \ln x] - [x]$$

7.) PI mit Trick (Pythagoras)

$$\int \sin^2 3x \, dx = \int \underset{u}{\sin 3x} \cdot \underset{v'}{\sin 3x} \, dx$$

$$= [\underset{u}{\sin 3x} \cdot \underset{v}{\tfrac{1}{3}(-\cos 3x)}] - \int \underset{u'}{3 \cdot \cos 3x} \cdot \underset{v}{\tfrac{1}{3}(-\cos 3x)} \, dx$$

$$= [-\tfrac{1}{3} \sin 3x \cdot \cos 3x] + \int \underbrace{\cos^2 3x}_{= 1 - \sin^2 3x} \, dx$$

$$\Leftrightarrow \quad 2 \int \sin^2 3x \, dx = [-\tfrac{1}{3} \sin 3x \cos 3x] + \int 1 \, dx$$

$$\Leftrightarrow \quad 2 \int \sin^2 3x \, dx = [-\tfrac{1}{3} \sin 3x \cdot \cos 3x + x]$$

$$\Leftrightarrow \quad \int \sin^2 3x \, dx = [-\tfrac{1}{6} \sin 3x \cdot \cos 3x + \tfrac{1}{2} x]$$

8.) Sub

$$\int x \cdot e^{-x^2} dx = -\frac{1}{2} \int -2x\, e^{-x^2} dx$$

$$z = g(x) = -x^2 \; ; \; g'(x) = -2x$$

$$f'(z) = e^z \; ; \; f(z) = e^z$$

$$= -\frac{1}{2}\left[e^{-x^2} \right]$$

9.) Sub

$$\int \cos x \cdot \sin^2 x \, dx = \left[\frac{1}{3} \sin^3 x \right]$$

$$z = g(x) = \sin x \, , \; g'(x) = \cos x$$

$$f'(z) = z^2 \quad f(z) = \frac{1}{3} z^3$$

10.) PI

$$\int \underset{u}{u} \cdot \underset{v'}{(u+7)^9}\, du = \left[\underset{u}{u} \cdot \underset{v}{\frac{1}{10}(u+7)^{10}} \right]$$

$$- \int 1 \; \frac{1}{10} (u+7)^{10} du$$

$$= \left[\frac{1}{10} u (u+7)^{10} \right] - \left[\frac{1}{110} (u+7)^{11} \right]$$

11.) Sub

$$\int \frac{x^2}{(x^3+9)^5} dx = \frac{1}{3} \int \frac{3x^2}{(x^3+9)^5} dx$$

$$z = g(x) = x^3 + 9 \; ; \; g'(x) = 3x^2$$

$$f'(z) = \frac{1}{z^5} \quad f(z) = -\frac{1}{4z^4}$$

12.) Sub

$$\int \frac{\sin 2y}{1-\cos 2y}\, dy = \frac{1}{2}\int \frac{2\sin 2y}{1-\cos 2y}\, dy$$

$z = g(x) = 1-\cos 2y \quad ; \quad g'(x) = 2 \cdot \sin 2y$

$f'(z) = \frac{1}{z} \quad ; \quad f(z) = \ln |z|$

$$= \frac{1}{2}\left[\ln |1-\cos 2y| \right]$$

13.) ...

$$\int \frac{1}{2x+7}\, dx = \left[\frac{1}{2} \ln |2x+7| \right]$$

14.) Sub.

$$\int \sin 3x \cdot \sqrt{1+\cos 3x}\, dx = -\int -\sin 3x \cdot \sqrt{\dots}$$

$z = g(x) = 1+\cos 3x \quad ; \quad g'(x) = -3\sin 3x$

$f'(z) = \sqrt{z} \qquad f(z) = \frac{2}{3} z^{3/2}$

$$= -\left[\frac{2}{3}(1+\cos 3x)^{3/2} \right]$$

15.) P. I

$$\int \underset{u}{x} \cdot \underset{v'}{\sin 4x}\, dx = \left[\underset{u}{x} \cdot \underset{v}{\frac{1}{4}(-\cos 4x)} \right]$$

$$- \int \underset{u'}{1} \cdot \underset{v}{\frac{1}{4}(-\cos 4x)}\, dx$$

$$= \left[-\frac{1}{4} x \cos 4x \right] + \left[\frac{1}{16} \sin 4x \right]$$

16.) $\int \frac{x+2}{x^2+4x-5}\, dx$

$$g(x) = x^2+4x-5 = z$$

$$(2x+4)\, dx = dz$$

$$dx = \frac{dz}{2x+4}$$

$$= \int \frac{x+2}{z} \cdot \frac{dz}{2x+4} = \int \frac{\cancel{x+2}}{z} \cdot \frac{dz}{2\cancel{(x+2)}}$$

$$= \int \frac{1}{2z}\, dz = \frac{1}{2} \int \frac{1}{z}\, dz$$

$$= \frac{1}{2} \left[\ln|z| \right] = \frac{1}{2} \left[\ln|x^2+4x-5| \right]$$

17.) wie 16.)

18.) $\int \frac{x+2}{(x^2+4x-5)^3}\, dx$

$$g(x) = x^2+4x-5 = z$$

$$(2x+4)\, dx = dz \Rightarrow dx = \frac{dz}{2x+4}$$

$$= \int \frac{\cancel{x+2}}{z^3} \cdot \frac{dz}{2\cancel{(x+2)}} = \frac{1}{2} \int z^{-3}\, dz$$

$$= \frac{1}{2} \left[-\frac{1}{2} z^{-2} \right] = \left[-\frac{1}{4} (x^2+4x-5)^{-2} \right]$$

19.) $\int 3y\sqrt{9-y^2}\,dy$

$$g(y) = 9-y^2 = z$$

$$-2y\,dy = dz$$

$$= \int 3\cancel{y} \cdot \sqrt{z} \cdot \frac{dz}{-2\cancel{y}} = -\frac{3}{2}\int z^{\frac{1}{2}}$$

$$= -\frac{3}{2}\left[\frac{2}{3} z^{3/2}\right] = \left[-(9-y^2)^{3/2}\right]$$

20.) $\int \underset{u}{e^{2x}} \cdot \underset{v'}{\cos 3x}\,dx = \left[e^{2x} \cdot \frac{1}{3}\sin 3x\right]$

$$- \int 2e^{2x} \cdot \frac{1}{3}\sin 3x\,dx$$

$$= [\ldots] - \frac{2}{3}\int e^{2x} \cdot \sin 3x\,dx$$

$$= [\ldots] - \frac{2}{3}\left(\left[e^{2x} \cdot \frac{1}{3}(-\cos 3x)\right] - \int 2e^{2x} \cdot \frac{1}{3}(-\cos 3x)\,dx\right)$$

$$= [\ldots] + \left[+\frac{2}{9}e^{2x}\cos 3x\right] - \frac{2}{3}\int e^{2x} \cdot \cos 3x\,dx$$

$$\Rightarrow \int e^{2x} \cdot \cos 3x\,dx = \left[e^{2x}\left(\frac{1}{3}\sin 3x + \frac{2}{9}\cos 3x\right)\right] - \frac{2}{3}\int \ldots$$

$$\Big| + \frac{2}{3}\int \ldots$$

$$\Leftrightarrow \frac{5}{3}\int e^{2x} \cdot \cos 3x\,dx = [\ \ldots\ ] \quad \Big| \cdot \frac{3}{5}$$

$$\Leftrightarrow \int e^{2x} \cdot \cos 3x\,dx = \left[\frac{3}{5}e^{2x}\left(\frac{1}{3}\sin 3x + \frac{2}{9}\cos 3x\right)\right]$$

21.) <u>wieder und</u>

$$\int \ln 5x \, dx = \int (\ln 5 + \ln x) \, dx$$

$$= \left[(\ln 5) \cdot x + \underbrace{x(\ln x - 1)}_{\text{siehe Aufgabe 6}} \right]$$

22.) P. I.

$$\int \cos^3 2x \, dx = \int \underset{v'}{\cos 2x} \cdot \underset{u}{\cos^2 2x} \, dx$$

$$= \left[\tfrac{1}{2} \sin 2x \cdot \cos^2 2x \right] - \int \tfrac{1}{\cancel{2}} \sin 2x \cdot \cancel{2} \cos 2x \cdot (-\sin 2x) \cdot 2 \, dx$$

$$= [\ldots] + 2 \int \cos 2x \cdot \sin^2 2x \, dx$$

$$= [\ldots] + 2 \int \cos 2x \, (1 - \cos^2 2x) \, dx$$

$$= [\ldots] + 2 \int \cos 2x - 2 \int \cos^3 2x \, dx$$

$$\Leftrightarrow \int \cos^3 2x \, dx = [\ldots] + 2 \left[\tfrac{1}{2} \sin 2x \right] - 2 \int \cos^3 2x \, dx$$

$$\Leftrightarrow 3 \int \cos^3 2x \, dx = [\ldots] + 2 \left[\tfrac{1}{2} \sin 2x \right]$$

$$\int \cos^3 2x \, dx = \tfrac{1}{3} \left[\tfrac{1}{2} \sin 2x \cdot \cos^2 2x + \sin 2x \right]$$

$$= \left[\tfrac{1}{6} \sin 2x \cos^2 2x + \sin 2x \right]$$

$$= \left[\sin 2x \left(\tfrac{1}{6} \cos^2 2x + 1 \right) \right]$$

23) Sub

$$\int \cos x \cdot e^{\sin x} dx$$

$$g(x) = \sin x = t$$
$$\cos x \, dx = dt$$
$$dx = \frac{dt}{\cos x}$$

$$= \int \cancel{\cos x} \, e^{t} \cdot \frac{dt}{\cancel{\cos x}}$$

$$= [e^{t}] = [e^{\sin x}]$$

24.) Sub

$$\int \frac{\sin y}{\sqrt{7 + \cos y}} dy$$

$$g(x) = 7 + \cos y = t$$
$$-\sin y \, dy = dt$$
$$dy = \frac{dt}{-\sin y}$$

$$= \int \frac{\cancel{\sin y}}{\sqrt{t}} \frac{dt}{-\cancel{\sin y}}$$

$$= -\int t^{-\frac{1}{2}} dt = \left[-2\sqrt{t}\right] = \left[-2\sqrt{7 + \cos y}\right]$$

25.) P. I

$$\int \underset{u}{x^2} \underset{v'}{e^x}\, dx = [\underset{u}{x^2} \cdot \underset{v}{e^x}] - \int \underset{u'}{2x} \cdot \underset{v}{e^x}\, dx$$

$$= [x^2 e^x] - \left([2x e^x] - \int 2e^x\, dx \right)$$

$$= [x^2 e^x] - \left([2x e^x] - [2e^x] \right)$$

$$= [(x^2 - 2x + 2) e^x]$$

26.) Sub

$$\int \frac{x}{x^2-4}\, dx$$

$$g(x) = x^2 - 4 = z$$

$$2x\, dx = dz$$

$$dx = \frac{dz}{2x}$$

$$= \int \frac{\cancel{x}}{z} \cdot \frac{dz}{2\cancel{x}} = \frac{1}{2} \int \frac{1}{z}\, dz = \frac{1}{2} [\ln |z|]$$

$$= [\tfrac{1}{2} \ln |x^2 - 4|]$$